Do the Coaches' Crashes and Their Usage Exposure Come from the Same Distributions? An Empirical Study Utilizing Wilcoxon and Kolmogorov-Smirnov Tests

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Research Article

Abstract

Since coaches' crashes have serious impacts on society, many attributes are studied as contributing factors related to the crashes. The present research work focuses on the existing statistical similarity between distribution functions of coaches' crashes and their usage exposure in the Iranian intercity road network. Traffic volume is considered as an important usage exposure for intercity buses/coaches over the intercity road network, so this study aims to check whether coaches' traffic volume and the frequency of their crashes come from the same distribution functions. The two-sample distribution function test of Wilcoxon as well as the goodness of fit test called Kolmogorov-Smirnov have been utilized to check the statistical similarity between the distribution functions of coaches' crashes and their usage exposure. Experimental data for coaches' traffic volumes and their crashes have been collected for three years, and the above-mentioned methods have been performed using the proportion form of collected data due to the existing their different scales. After performing two methods of checking similarity, the results revealed there is no significant difference between the probability functions of traffic volumes, considered as usage exposure, and coaches' crashes. It means that both attributes come from the same distribution functions. Traffic safety authorities and passenger transport operators who are dealing with traffic safety on the intercity public should be aware of the effects of traffic volumes on coaches' crashes on designing the routes and schedule for passenger transportation.

Keywords: Similarity of Distribution Functions, Road Safety, Coach Crashes, Usage Exposure, Traffic Accidents' Contribution Factors.

1. Introduction

There is no week when nothing is heard about a happening a coach crash or bus accident across the Iranian intercity road network. Road transport operators carry more than 90 percent of total public transport passengers in the country where rail and aviation are not sufficiently established comparing to the existing inland transport infrastructures (RMTO, 2019). In this situation, many coaches' crashes (rollover, collisions, and hitting to fixed obstacles) have happened over the intercity road network which connects the most remote areas to the national road network. Many passengers feel more comfortable to sleep on the seat while making an intercity trip, so passenger transport operators are planning the routes and bus scheduled time trips over the night. Some experts and transport authorities believe the coaches' traffic volume is the leading contributing factor and consequently, it is naturally acceptable if the rate of night-time accidents would be higher than the daytime. This assumption remains such that the accident frequency of coaches depends on the particular exposure of traffic volume because the coaches' scheduled time and routes are usually proposed to set over the night. On the other hand, bus driver fatigue is known as the main factor on transport safety leading to banning transport operators to design their trips at early-hour of days in particular where the fundamental issues, such as driving and working hours, and the need for rest and sleep facilities for drivers, are not addressed (Mohd-Yusoff et al., 2012). While drivers who suffer from fatigue in long-distance trips increase the likelihood of a serious accident, safety policies such as restricting drivers from exceeding the limit of daily driving hours, installing an automatic sleep-warning device in the vehicle, testing drivers with obstructive sleep apnea syndrome or sleep disorders, educating the public or even amending the seatbelt legislation have been recommended to promote transport public transport (Chu, 2014). Therefore, the other assumption lies in the fatigue as the leading contributing factor in which night time is more likely to be risky for all drivers. As the novelty, the aim is to look at the dependency of traffic variables according to their distribution functions. To check the first mentioned assumption from another view, evidence-based research work using experimental data should be conducted utilizing the statistical methods of checking similarity between coaches' crashes and their usage exposure which is now limited to traffic volume due to data availability. In this research work, two well-known statistical methods of Wilcoxon and Kolmogorov-Smirnov tests are performed to conduct the required study where two distribution functions of coaches' crashes and traffic volume during day and night times will be compared in terms of the existing of statistical similarity.

1.1. Coaches' crashes and attributes

A view upon coaches' crashes within recent years indicates a dramatic figure of high frequency and severity outcomes in Iran. Based on domestic regulation (Khademi, & Choupani, 2018), Road Maintenance and Transport Organization analyze annual coaches' accidents to report to the National Road Safety Council every year. As shown in figure 1, on average more than 10 crashes were reported each month, shown on the left vertical axle by annual scale, whether the coach driver was the culprit or just involved in a sudden impact. What captures further attention is the coach crash rate that however was reduced through the precedent years (from 9 in 2012 to 6.3 in 2019), but the amount is considerably higher than the total crash rate shown on the right vertical axle. This is a social belief in which the coaches as a public transport mode must be considerably safer than traveling by private cars. That is why whenever a coach crash involving is reported, the society takes a negative position for using coaches in long distances, although the medium income of bus commuters typically has not left any other substitution for coaches not only in Iran but in other developing countries (Carruthers et al., 2005). For this reason, the coach trip number has not been changed tremendously within recent years, and consequently, further exposures and more crashes were recorded.

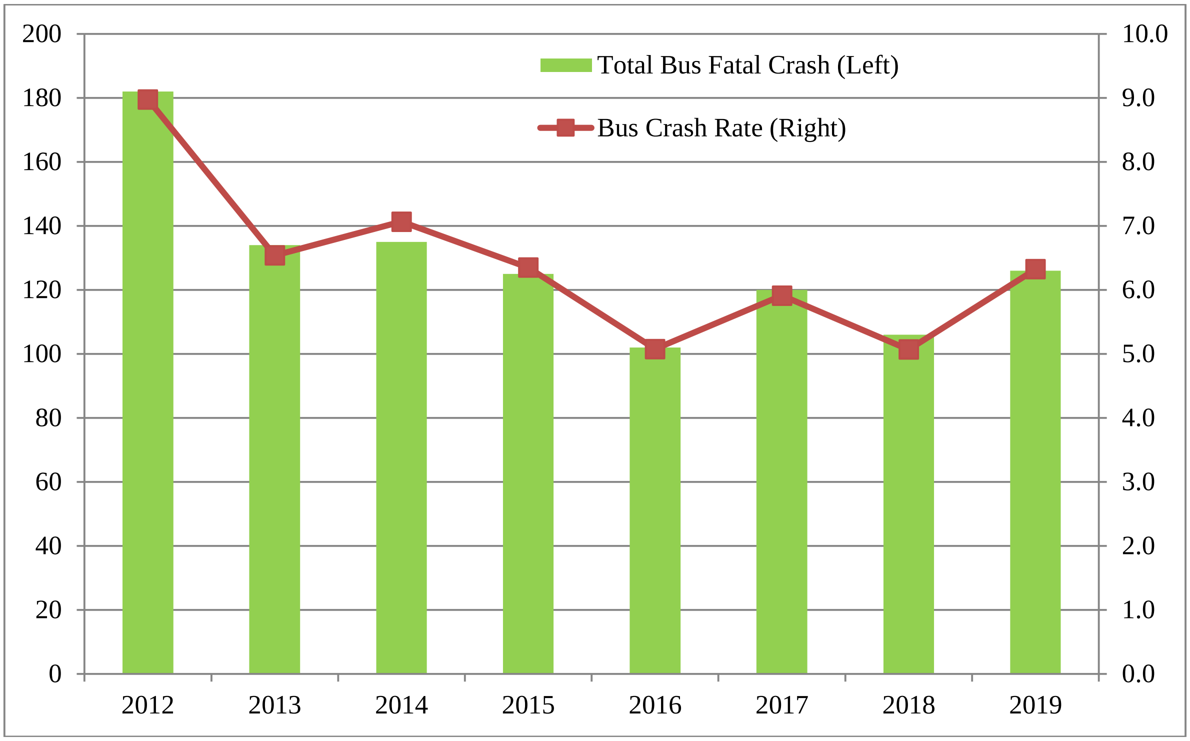

Figure 1: Coaches' Crash Frequency and Crash Rate across the Intercity Road Network of Iran

Although many risk factors are studied on the severity of coaches' crashes like the season, day of the week, time, the number of vehicles involved, land use, manner of the collision, speed limit, snow or ice surface condition, school bus, bus type, and seating capacity, driver's age, driver's gender, risky behaviors and restraint system (Feng et al., 2016), but the frequency or the rate of crashes regardless to severity accidents should be studied because of the social expectations of public transport safety. In addition, the influence of demographic, environmental, and occupational factors, and behavioral characteristics of intercity bus drivers have been also examined on their crash involvement risk, and results in Iran revealed that hazard monitoring, fatigue proneness, and thrill-seeking might be considered other significant predictors of crash involvement risk as well (Besharati & Tavakoli Kashani, 2018).

The independency of bus traffic accidents to the previous ones has also been studied in the literature where it was concluded that previous accident type affects the duration of the upcoming traffic accident while accident severity decreased and the time between two accidents increased when the previous accident was severe (Hamed et al., 1998). Nevertheless beside exposure other factors such as travel time, infrastructure quality, risk-taking or tiredness of bus drivers, etc. are contributing to coach crash occurrence as well (Copsey et al., 2010). Therefore in the case of preventing such crashes, transport authorities should be essentially supported by undertaking a holistic analysis of crash data in addition to the facts coming from in-depth crash investigations.

1.2. Distribution Similarity and concepts

Checking the similarity of distribution functions is one of the well-known methods of examining the relevancy of variables. In this case, two or more data sets are compared based on the similarity of their distribution functions utilizing statistical tests (Pastore & Calcagnì, 2019). Two tests of Wilcoxon Signed-Rank and Kolmogrove-Smirnove are primarily utilized in non-parametric hypothesis testing (Şahinturk & Özcan, 2017). The essential concept behind the above tests is to compare the behavior of two related samples in which each record is compared individually to the same observation on the other population. This is named the paired difference test which means that each record has two data sets in which their differences are compared.

The Kolmogorov–Smirnov test (K–S test or KS test for short) is also a nonparametric test of the equality of continuous or discrete, one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution or compare two samples (Vrbik, 2018). The main concept behind the test hypothesis is to quantify the distance between the cumulative distribution functions of two samples as well as the similarity between the two shapes. It is also utilized to assess the similarity between the expected distribution function and experimental one which provides a practical tool to assess the fitness of the experimental data to the expected distribution function (Simard & Lecuyer, 2011). This ability usually supports data analyzers for normality testing where normal distribution function is assumed to perform data analysis procedures but it can be utilized in other distribution functions and existing similarities for two data sets.

The Wilcoxon signed-rank test is a non-parametric statistical hypothesis test used to compare two related or matched samples, or repeated measurements on a single sample to assess whether their populations mean ranks differ or not (Wilcoxon Frank, 1945). It is a nonparametric test that can be used to determine whether two dependent samples were selected from populations gaining from the same distribution. While the zero difference observations are discarded trough calculation procedure mainly in discrete values, but the test can be reasonably utilized without any concern for comparing continuous values (Derrick & White, 2017).

2. Data collection

To check the similarity between traffic volumes and coach accidents, data has been collected over the day and night times and sorted out by daily hours as well. All coaches' crashes data reported from 21st March 2017 to 20th March 2019 (1396-1398 in the Iranian local calendar) have been extracted from a database available in Road Maintenance and Transport Organization in Iran. Although many attributes may be contributed in usage exposures, the more reliable one is traffic volume which is electronically sensed and gathered over the intercity road network traffic coaches extracted by plate recognition devices installed across the roads. Data for all types of coaches' crashes including rollover, collision, and hitting to obstacles have been collected as well as hourly traffic volumes of coaches have been also gathered by traffic counters installed over the road network. Table 1 demonstrates traffic and accident data of coaches' crashes sorted into 24 hours of the daytime.

The first column indicates the time interval (divided into 24 hours a day). The second is the average number of coaches detected by the license plate cameras are under the usage over the intercity road network for traffic counting and other purposes of transport planning. The value represents the average number of coaches detected over the specific time period. As a result, it is a mean value used as the coaches' usage exposure over the network. The third is the number of all coaches' crashes that happened over the past three years. Since the scales are different, their proportion is calculated to uniform or standardize data for modeling in the closed interval of (0-1). More on how to rescale data is available at (Jongman, et al., 1995).

Table 1: Average traffic volume and coaches' crashes over the past three years

Time Interval	Raw Data		Proportion		Cumulative Probability		
	Traffic	Crashes	Traffic	Crashes	Traffic	Crashes	Abs(Diff)
00:00-00:59	2334794	26	0.0450	0.0358	0.0450	0.0358	0.0092
01:00-01:59	2525816	31	0.0487	0.0427	0.0937	0.0785	0.0152
02:00-02:59	2507696	29	0.0483	0.0399	0.1420	0.1185	0.0236
03:00-03:59	2539360	48	0.0490	0.0661	0.1910	0.1846	0.0064
04:00-04:59	2589410	50	0.0499	0.0689	0.2409	0.2534	0.0125
05:00-05:59	2104578	77	0.0406	0.1061	0.2815	0.3595	0.0780
06:00-06:59	2348190	55	0.0453	0.0758	0.3267	0.4353	0.1085
07:00-07:59	2139602	32	0.0412	0.0441	0.3680	0.4793	0.1113
08:00-08:59	1806672	30	0.0348	0.0413	0.4028	0.5207	0.1178
09:00-09:59	1694264	20	0.0327	0.0275	0.4355	0.5482	0.1127
10:00-10:59	1628504	22	0.0314	0.0303	0.4669	0.5785	0.1116
11:00-11:59	1670100	18	0.0322	0.0248	0.4991	0.6033	0.1042
12:00-12:59	1657508	21	0.0320	0.0289	0.5310	0.6322	0.1012
13:00-13:59	1717780	17	0.0331	0.0234	0.5641	0.6556	0.0915
14:00-14:59	1997594	14	0.0385	0.0193	0.6026	0.6749	0.0723
15:00-15:59	2292568	16	0.0442	0.0220	0.6468	0.6970	0.0501
16:00-16:59	2610144	34	0.0503	0.0468	0.6972	0.7438	0.0467
17:00-17:59	2536832	26	0.0489	0.0358	0.7461	0.7796	0.0336
18:00-18:59	2535660	30	0.0489	0.0413	0.7949	0.8209	0.0260
19:00-19:59	2470280	19	0.0476	0.0262	0.8426	0.8471	0.0046
20:00-20:59	2118366	27	0.0408	0.0372	0.8834	0.8843	0.0009
21:00-21:59	1937058	23	0.0373	0.0317	0.9207	0.9160	0.0048
22:00-22:59	1965742	25	0.0379	0.0344	0.9586	0.9504	0.0082
23:00-23:59	2146318	36	0.0414	0.0496	1.0000	1.0000	0.0000

3. Hypothesis Testing

Two well-known methods of similarity testing including Kolmogorov-Smirnov and Wilcoxon are utilized in the present research work. In this case, the null and competitive hypothesis will be defined as follows:

H0: Coaches' crashes and their usage exposure (traffic volumes) distributions are the same.

H1: Coaches' crashes and their usage exposure (traffic volumes) are significantly different.

As said in advance, the rescaled amount of each parameter is additionally used for calculation processes. Assume Xt is the proportion of coaches' traffic volume, Yt is the proportion of coaches' crashes, both defined in time interval t which covers day and night times from 1 to 24. Stats for both tests will be calculated and compared to the critical values for each test. If the calculated values are less than the critical ones, the null hypothesis will be accepted which means that traffic and crashes variables come from the same distributions, otherwise the null hypothesis will be rejected or their distributions are significantly different.

3.1. Performing Kolmogorov – Smirnov Test

To perform the Kolmogorov - Smirnov test, the cumulative probabilities for all time intervals should be calculated and followed by determining the maximum value of difference. In this case, traffic volume is considered as the expected and the crash as the experimental probability. The last three columns of table 1 represent the calculated values as well as figure 2 depicts them by continuous line for traffic volume and dashed line for crashes cumulative probabilities. For example; in the time interval 01:00 – 01:59, where t=2, the cumulative probability of traffic volume (Xt) is calculated as 0.0450 + 0.0487 = 0.0937 followed by the same way for crashes (Yt) calculated as 0.0358 + 0.0427 = 0.0785. The absolute difference between expected and experimental cumulative probabilities is now calculated as |0.0937-0.0785| = 0.0152 tabulated in the last column.

The previous process has been performed for all time intervals and the maximum difference, called K-S stat, is now determined as 0.1178 for (t=9) demonstrated by a circle in figure 2. It should be compared to K-S(95%, 24) = 0.269 known as the critical value for Kolmogorov - Smirnov test (D'Agostino 1986). As observed, the calculated stat of Dmax (= 0.1178) is less than the critical value of the K-S test (= 0.269), so it means that coaches' traffic volume and their crashes frequency come from the same distributions.

3.2. Performing the Wilcoxon Test

To utilize the Wilcoxon test, the sign function is performed for all time intervals where the rescaled amount of traffic volumes and crashes are respectively shown by variables Xt and Yt. Table 2 shows all calculated values performing the Wilcoxon test. It is divided into two parts; original values set on the left side as well as ordered values in the right. For example, in time interval 00:00-00:59 (t=1); the value of (X1-Y1= 0.0450-0.0358) is equal to 0.0092. If it is greater than zero, the sign value is equal to 1 while for negative values is equal to -1. If the value is negative, the absolute value will be set in the positive form. The ordered values (in absolute) are shown on the right side in which the minimum and the maximum are respectively are 0.0011 and 0.0655.

The ranks for all differences are also set on the fifth column on the right side followed by determining the sign multiply to the corresponding rank on the last column. The value of W is calculated by equation (1) (Ghosh, 1984) followed by calculating δ_w based on equation (2) (Csorgo & Horvath, 1988) where the number of observations is 24. The normal standard value of Z is eventually determined by equation (3) because the number of observations exceeds 20 (Csorgo & Horvath, 1988).

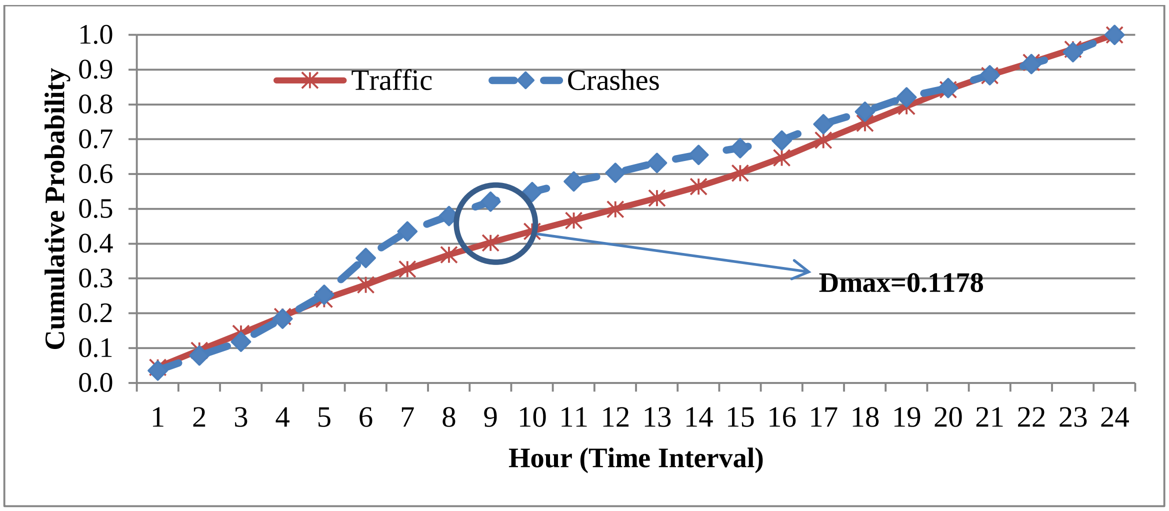

Figure 2: Difference between cumulative probabilities of Coaches' Traffic Volume and Crashes

$$W = \sum_{t=1}^{24} [sgn(X_t - Y_t) \times R_t] = 1 + (-2) + 3 + \ldots + (-24) = 82 \quad (1)$$

$$\sigma_w = \sqrt{\frac{N(N+1)(2N+1)}{6}} = \sqrt{\frac{24(24+1)(2 \times 24 + 1)}{6}} = 70 \quad (2)$$

$$Z = \frac{W}{\sigma_w} = \frac{82}{70} = 1.171 \quad (3)$$

The Wilcoxon stat, which is determined as 1.171, should be compared to the critical value of the normal standard distribution of Z(0.975,0,1)=1.96 with a confidence interval of 95% (Csorgo & Horvath, 1988). As observed, Z stat is less than the critical value means that traffic and crashes for intercity coaches come from the same distributions.

The smooth differences of two variables (coaches' crashes and usage exposures) indicate that the leading contributing factor of coaches' crashes may come from their usage exposure studied by traffic volumes over the intercity road network.

4. Summary and Conclusion

The dependency between the coaches' usage exposure (traffic volumes) and their crash frequencies on the intercity road network has remained a primary concern for authorities who are dealing with public transport safety. To gain an understandable response on whether usage exposure of coaches and their crash frequencies come from the same distribution functions, data on traffic volumes (represent the usage exposure) for coaches and their crashes have been gathered for three years. One of them (traffic volume & crash frequency) is designated as the expected and the other as the experimental variable followed by utilizing two well-known methods of Wilcoxon and Kolmogorov - Smirnov tests for checking similarity between distribution functions.

Table 2: Statistical measures for proportions of coaches' traffic volumes and crashes

Original values and differences						Ordered by an absolute value						
t	Xt	Yt	Xt - Yt			t	Xt	Yt	Xt - Yt			
			Value	Sign	Absolute				Sign	Absolute	Rt	Sign.Rt
1	0.0450	0.0358	0.0092	1	0.0092	11	0.0314	0.0303	1	0.0011	1	1
2	0.0487	0.0427	0.0060	1	0.0060	8	0.0412	0.0441	-1	0.0028	2	-2
3	0.0483	0.0399	0.0084	1	0.0084	13	0.0320	0.0289	1	0.0030	3	3
4	0.0490	0.0661	-0.0172	-1	0.0172	23	0.0379	0.0344	1	0.0035	4	4
5	0.0499	0.0689	-0.0190	-1	0.0190	17	0.0503	0.0468	1	0.0035	5	5
6	0.0406	0.1061	-0.0655	-1	0.0655	21	0.0408	0.0372	1	0.0036	6	6
7	0.0453	0.0758	-0.0305	-1	0.0305	10	0.0327	0.0275	1	0.0051	7	7
8	0.0412	0.0441	-0.0028	-1	0.0028	22	0.0373	0.0317	1	0.0057	8	8
9	0.0348	0.0413	-0.0065	-1	0.0065	2	0.0487	0.0427	1	0.0060	9	9
10	0.0327	0.0275	0.0051	1	0.0051	9	0.0348	0.0413	-1	0.0065	10	-10
11	0.0314	0.0303	0.0011	1	0.0011	12	0.0322	0.0248	1	0.0074	11	11
12	0.0322	0.0248	0.0074	1	0.0074	19	0.0489	0.0413	1	0.0076	12	12
13	0.0320	0.0289	0.0030	1	0.0030	24	0.0414	0.0496	-1	0.0082	13	-13
14	0.0331	0.0234	0.0097	1	0.0097	3	0.0483	0.0399	1	0.0084	14	14
15	0.0385	0.0193	0.0192	1	0.0192	1	0.0450	0.0358	1	0.0092	15	15
16	0.0442	0.0220	0.0222	1	0.0222	14	0.0331	0.0234	1	0.0097	16	16
17	0.0503	0.0468	0.0035	1	0.0035	18	0.0489	0.0358	1	0.0131	17	17
18	0.0489	0.0358	0.0131	1	0.0131	4	0.0490	0.0661	-1	0.0172	18	-18
19	0.0489	0.0413	0.0076	1	0.0076	5	0.0499	0.0689	-1	0.0190	19	-19
20	0.0476	0.0262	0.0214	1	0.0214	15	0.0385	0.0193	1	0.0192	20	20
21	0.0408	0.0372	0.0036	1	0.0036	20	0.0476	0.0262	1	0.0214	21	21
22	0.0373	0.0317	0.0057	1	0.0057	16	0.0442	0.0220	1	0.0222	22	22
23	0.0379	0.0344	0.0035	1	0.0035	7	0.0453	0.0758	-1	0.0305	23	-23
24	0.0414	0.0496	-0.0082	-1	0.0082	6	0.0406	0.1061	-1	0.0655	24	-24

Using experimental data and calculating stats obtaining by utilizing the statistical methods followed by comparing the stats and critical values, a summary result on what has been calculated and critical values extracted from statistical tables have been tabulated in Table 3 where the last column makes the concluding remarks. As shown in table 3, the calculated stats are less than the critical values for both comparing methods, so the results revealed that the coaches' usage exposure and their crashes frequency come from the same distribution functions. As an implication note, it should be mentioned that traffic volume plays a significant role in happening coaches' crashes over the Iranian intercity network. So traffic safety authorities and transport passenger operators should be aware of the effects of traffic volume on public safety for coaches. They can think of some interventions to uniform traffic volumes over the daytime but at the same time should be aware of satisfying the transport demands in which passengers are mainly interested in making their trips. Further researches are recommended to check the similarity functions on traffic volumes not only for coaches but also for other vehicles to indicate that if total traffic volume and coaches' crashes come from the same distribution functions.

Table 3: Summary results on the similarity of coaches' usage exposure and crashes

Method of Test	Measure	Stat	Critical value	Conclusion
Kolmogorov-Smirnov	Dmax	0.1178	0.296	Coaches' usage exposure and crash frequency come from the same distribution functions.
Wilcoxon	$\frac{W}{\sigma_w}$	1.171	1.96	Coaches' usage exposure and crash frequency come from the same distribution functions.

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Conflict of Interest: The authors declare no conflict of interest.

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