A Multi-Objective Mathematical Model for Vehicle Routing Problem Considering the Time Window and Economic and Environmental Objectives Using the Metaheuristic Algorithm Based on Pareto Archive

Elnaz Asadifard¹, Maryam Adlifard², Mohammad Taghipour³*, Nader Shamami⁴

¹Industrial Engineering, Ghiaseddin Jamshid Kashani, Non-Profit University, Qazvin, Iran
²Department of Mathematics, Roudbar Branch, Islamic Azad University, Roudbar, Iran
³Young Researchers and Elites Club, Science and Research Branch, Islamic Azad University, Tehran, Iran
⁴Engineering-Operations Research and Systems Engineering of Qazvin, Islamic Azad University Qazvin, Iran

*Corresponding Author: mohamad.taghipour@srbiau.ac.ir


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Research Article

Abstract

Purpose: The purpose of this study is the well-known Heterogeneous Fleet Vehicle Routing Problem (HVRP) is one of the developed problems of vehicle routing, which involves optimizing a set of routes for a fleet of vehicles with different costs and capacities.

Methods: HVRP is usually modeled as a single objective that aims to minimize overall transportation costs (total fixed costs and costs commensurate with total distance).

Results: These vehicles are located in the central depot and serve customers’ demands.

Implications: In this case, the number of vehicles available (of any type) may be limited.

Keywords: vehicle routing, multi-objective mathematical model, metaheuristic algorithm.

1. Introduction

Studies report that demand for transportation and freight is growing rapidly. Transportation also accounts for 24% of greenhouse gas emissions in Europe. Road transportation of people and goods is the main source of greenhouse gas emissions (about 17%). In this regard, the European Union (EU) has set standards to limit the emission of pollutants by new vehicles in Europe since 1993. Also, a framework was designed to reduce transport-related pollution in
Europe by about 60% by 2050 compared to 1990. In this context, the energy and environmental aspects are very important and there is a need to develop low-cost logistics policies by considering the environmental components in the decision-making process. These policies should be limited to energy aspects. Heterogeneous Fleet Vehicle Routing Problem (HVRP) can now be extended by adding objective functions that minimize waiting times as well as emissions of greenhouse gases and pollutants. Also, the problem can be modeled as a multi-objective optimization to develop a mixed-integer linear programming (MILP) problem and minimize internal and waiting costs, as well as greenhouse gas emissions. This makes it possible to consider the social and economic aspects as well as issues such as energy and environment in modeling HVRP.

2. Literature review

Azarian, Tizfahm, Habibi Machiyani, & Taghipour (2020) studied the effect of implementing total quality management on job satisfaction (A case study). The results showed a significant difference between the implementation of TQM and increasing job satisfaction in the education organization. Alamdar Khoolaki, Naami, & Taghipour (2019) studied the Effect of integrated marketing communication on brand value with the role of the agency’s reputation. Taghipour, Mahboobi, Nikoeifar, & Soofi Mowlodii (2015) studied Analyzing the Effects of Physical Conditions of the Workplace on Employees Productivity. HVRP was first raised by Taillard in 1999 (Tan, Chew, & Lee, 2006). Since then, many studies have been done to find a solution to this problem. In 2006, for example, Tan et al. introduced the problem of dual-purpose vehicle routing considering the time window. The objective functions in this problem were to minimize the number of vehicles and the distance traveled (Taillard, 1999). Jozefowiez, Semet & Talbi (2009) propounded the problem of dual-purpose vehicle routing considering capacity limitation the objective functions of which were to minimize the total distance traveled, and the number of unbalanced routes (Jozefowiez et al, 2009). In recent years, with green logistics and environmental issues, several studies on vehicle routing issues have also considered environmental issues in their target functions. Kara, Kara, & Yetis (2007) proposed a model based on the vehicle routing problem in which a weight function of the loads is minimized to reduce energy consumption (Kara et al, 2007). Figliozzi (2010) proposed the Emission Vehicle Routing Problem (EVRP). In this problem, the number of pollutant emissions was considered as a function of travel speed and travel time, and limitations of time windows and capacity were also applied to the problem (Figliozzi, 2010). Maden, Eglese, & Black (2010) raised the problem of Vehicle Routing and Scheduling Problem (VRSPD). They considered time windows in which speed depended on travel time (Maden et al, 2010). Pollutant Routing Problem (PRP) was raised by Bektas and Laporte in 2011 to measure and minimize the cost of greenhouse gas emissions (Bektaş & Laporte, 2011). Jabali, Woensel, & Kok (2012) also raised the VRSP, in which they estimated the number of harmful gases emitted by vehicles and estimated it based on a nonlinear function of speed and some other factors, they finally developed a model that finds the optimal speed by considering the emission rate of pollutants (Jabali, 2012). Xiao, Zhao, Kaku, & Xu (2012) raised the problem of Fuel Consumption Rate (FCR). This rate is a function of
the amount of load carried by the vehicle added to the classic vehicle routing problem and the objective function of minimizing fuel consumption was also considered (Xiao et al, 2012. Eguia, Racero, Molina, & Guerrero (2013) considered a mathematical programming model for the HVRP problem with a time window and backhauls in which the external costs were internalized. Siu, Chan, & Chan (2012), proposed a multi-objective vehicle routing problem, one of the functions of which was to minimize greenhouse gas emissions. In 2014, Molina et al. presented a multi-objective vehicle routing problem that had three objective functions, the objective functions dealt with minimizing the internal costs of the transportation system and minimizing the emission of air pollutants and greenhouse gases.

3. The innovation of the project
Much research has been done on the problem of multi-objective vehicle routing which is related to the HVRP. However, so far, no HVRP problem has addressed the economic, customer satisfaction, and environmental aspects simultaneously. Therefore, the main innovation of the project is to consider these three aspects, which can be specifically considered in the discussion of early and late service time. This consideration adds a kind of time window constraint to the main problem. In the following, we describe the objectives considered in each aspect of the problem.

3.1 The economic aspect of the problem
- Minimizing the internal costs of the transportation network: these costs include drivers' wages, fuel costs, and fixed costs (such as purchasing vehicles) of the network.
- Minimizing the lost opportunity cost: this is the cost of vehicles arriving at customers' locations earlier than the expected time. This is because if the vehicle arrives at the destination early, the customer is not necessarily ready to receive the service. Therefore, it has to bear the waiting time, which is costly and a waste of opportunity.

3.2 The customer orientation aspect of the problem
- Minimizing the cost of lost credit: if the vehicle arrives late, the customer may be offended. So we have to bear the cost of the lost credit.

3.3 The environmental aspect of the problem
- Minimizing the emission of air pollutant gases
- Minimizing greenhouse gas emissions

4. Problem definition and modeling
In this project, it is assumed that CO2 is the only greenhouse gas emitted by transportation (excluding methane and nitrate oxide). It is also assumed that all the carbon in the fuel is converted to CO2. The coefficient emission factor can be considered for each unit of fuel to determine the amount of CO2 emitted per unit of fuel consumed. Fuel consumption depends on three factors: the distance traveled, the type of vehicle, and the cargo carried. In addition to
greenhouse gases, several gases also cause air pollution. In this project, NOx is assumed as the only gas emitted this way. A coefficient can also be considered based on the distance traveled to determine the amount of pollution unit (e.g.: grams) in terms of units of distance traveled (e.g.: kilograms). It should be noted that in this project it is assumed that different vehicles consume different fuels. In the following, we will model the problem: The HVRPTWj can be defined on the graph \( G = (N, A) \) where \( N = \{0,1,..., n\} \) is equal to the set of nodes that node zero represents the depot and other nodes represent the delivery locations. Set \( A \) also represents the set of graph arcs. We represent the set of the available heterogeneous vehicle (m) as \( Z = \{1,2,..., m\} \).

To model the problem, we define the following symbols:

- \( D_i \): The amount of freight demand by node \( i (i \in \{1,2,..., n\}) \)
- \( q^k \): Capacity of vehicle \( k (k \in \{1,2,..., m\}) \)
- \( e_i \): The earliest time it takes for a service vehicle to reach node \( i \)
- \( l_i \): The latest time it takes for a service vehicle to reach node \( i \)
- \( CO_i \): The missed opportunity cost due to the imposition of waiting time due to early arrival of a vehicle to service at node \( i \)
- \( CR_i \): Cost of lost credit due to the late arrival of the vehicle to service node \( i \)
- \( s^k \): The service time of vehicle \( k \) in node \( i \)
- \( d_{ij} \): The distance between node \( i \) and node \( j \)
- \( t_{ij} \): Time required to drive from node \( i \) to node \( j \) (\( i \neq j \))
- \( T^k \): Maximum driving time for vehicle \( k \)
- \( x^k_{ij} \): This is a variable of zero and one whose value is 1 when vehicle \( k \) travels from node \( i \) to node \( j \) (\( i \neq j \)), otherwise its value is zero.
- \( y^k_i \): The variable of the decision to start service at node \( i \) by vehicle \( k \) (\( y^k_i \) is the end time)
- \( f^k_{ij} \): The decision variable representing the amount of load transferred by vehicle \( k \) from node \( i \) to node \( j \) (\( i \neq j \)).
- \( p^k \): The wage of the driver of vehicle \( k \) per unit time
- \( f^r \): Cost of fuel type \( r \)
- \( f^e_k \): Fuel consumption for empty vehicle \( k \)
- \( f^e_{eu} \): Fuel consumption for empty vehicle \( k \) for each amount of load added
- \( \delta^{kr} \): This is a variable of zero and one whose value is 1 when the vehicle \( k \) uses type \( r \) fuel
- \( f^x_k \): Fixed costs of vehicle \( k \)
- \( mn^k \): Costs of maintenance of vehicle \( k \) per kilometer
- \( tl_{ij} \): Arc-related toll costs (\( i, j \))
- \( ef^{ CO_2 r} \): Emission factor for fuel type \( r \)
- \( ef^{p} \): The amount of pollution \( p \) emitted from a vehicle equipped with technology \( t \) per kilometer traveled
$y^{kt}$: This is a variable of zero and one whose value is 1 when the vehicle $k$ is equipped with technology $t$.

We now model the problem according to propositions (1) to (15).

\[
F_1(x, y, f) = \min \sum_{k=1}^{m} p^k y^k_0 + \sum_{i=0}^{n} \sum_{k=1}^{m} \sum_{r=1}^{r} f^{kr} \delta^{kr} d_{ij}(k x^k_{ij} + f e^k f^k_{ij}) \\
+ \sum_{i} \sum_{k} f^{kr} x^{k}_{il} + \sum_{i} \sum_{j} \sum_{n} \sum_{m} m n^{k} d_{ij} x^{k}_{ij} + \sum_{i} \sum_{j} \sum_{p} t_{ij} x^k_{ij} 
\]

\[
F_2(y) = \min \sum_{i=0}^{n} \sum_{k=1}^{m} C O_i (e_i - y^k_i)^+ 
\]

\[
F_3(y) = \min \sum_{i=0}^{n} \sum_{k=1}^{m} C R_i (y^k_i - l_i)^+ 
\]

\[
F_4(x, f) = \min \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=1}^{m} \sum_{r=1}^{r} \delta^{kr} e^{c02, r} d_{ij}(f e^k x^k_{ij} + k f^k_{ij}) 
\]

\[
F_5(x) = \min \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=1}^{m} \sum_{r=1}^{r} \sum_{p=1}^{p} \delta^{kr} x^{kt} e^{p, t} d_{ij} x^k_{ij} 
\]

\[
\sum_{j=1}^{n} x^k_{ij} \leq 1 \quad (k = 1, \ldots, m) 
\]

\[
\sum_{j=0}^{n} x^k_{ij} - \sum_{j=0}^{n} x^k_{ji} = 0 \quad (k = 1, \ldots, m \ , \ i = 1, \ldots, n) 
\]

\[
\sum_{k=1}^{m} \sum_{j=0}^{n} x^k_{ij} = 1 \quad (i = 1, \ldots, n) 
\]

\[
\sum_{i=1}^{n} D_i \sum_{j=0}^{n} x^k_{ij} \leq q^k \quad (k = 1, \ldots, m) 
\]

\[
y^k_i + s^k_i + t_{ij} \leq y^k_j + T^k(1 - x^k_{ij}) \quad (i = 1, \ldots, n, \ j = 0, \ldots, n, \ j \neq i, k = 1, \ldots, m) 
\]

\[
t^k_0 \leq y^k_j + T^k(1 - x^k_{0j}) \quad (j = 1, \ldots, n, \ k = 1, \ldots, m) 
\]

\[
y^k_0 \leq T^k \quad (k = 1, \ldots, m) 
\]
In this modeling, limit (6) indicates that each vehicle starts moving from the depot (or does not move at all). Limit (7) states that any vehicle that leaves a node also passes through that node when it returns. Limit (8) ensures that each customer and each supplier is met only once by one vehicle. Limit (9) ensures that none of the devices are overloaded. Service start times are calculated in limits (10) and (11). Limit (12) does not allow driving out of the allowed time. In the limit (13) the transport flow is balanced. Limits (14) and (15) are used to limit the overall load of a vehicle. The answers to the routing problem must be under the five objective functions (1), (2), (3), (4), and (5). The objective function (1) is related to minimizing the internal costs of the transportation system (driver costs, fuel). The objective function (2) is to minimize missed opportunity costs due to vehicles arriving earlier than the expected time. The objective function (3) is to minimize the cost of lost credit due to the late arrival of vehicles to customers’ locations. The objective function (4) is related to minimizing greenhouse gas emissions (here: CO2) and objective function (5) is related to minimizing emissions of air pollutants (here: NOx). It should be noted that the objective functions (2) and (3) are somehow related to the limits of the time windows, which are expressed as objective functions.

5. Proposed methods

5.1 Vector evaluated genetic algorithm (VEGA)
This is one of the first methods that used a genetic algorithm to solve a multi-objective problem. In each iteration of this algorithm, the population of answers is divided into n (number of objective functions) subset. These subpopulations are combined by the neighbor operators of genetic algorithms to form a smaller population (Jozefowiez et al., 2008).

5.2 Scattered search algorithm to solve multi-objective functions
A scattered search algorithm is a population-based algorithm and uses a set that contains quality and scattered answers (reference set) to combine them to create other answers. First, a population of initial answers is placed in the reference set, and at each step, according to the quality of the answers, the reference set is updated until we reach the condition that the algorithm stops. To implement the scattered search algorithm, researchers have modified the reference set so that the quality of the answers within it is evaluated by all objective functions. Also, the dispersion of the answers is equivalent to finding the answers that cover the Pareto efficiency boundary. This makes the algorithm have two important properties. The first is that the answers found converge to the Pareto optimal points (empirically) and the second is that the answers found are scattered along the Pareto boundary (Melián-Batista, De Santiago,
Angelbello, & Alvarez (2014)). For example, Figure (1) shows the solving of the bi-objective scatter search algorithm (BIOSS).

```
1 repeat
2     // Create the Initial Population, Pop
3     while (|Pop| < PopSize or a maximum number of iterations is not reached) do
4         Construct a feasible solution S;
5         Apply the Local Searches in the following order:
6             S¹ ← LocalSearchDistance(S);
7             S² ← LocalSearchTimeBalance(S¹);
8             S³ ← LocalSearchDistance(S²);
9             S⁴ ← LocalSearchCompromise(S³);
10            Pop ← Pop ∪ {S⁴};
11     // Generate the Efficient Set, Ŕ, and run the remaining steps of Scatter Search
12     while (new solutions are added to the EfficientSet or a maximum number of iterations is not reached) do
13         Generate or Update the EfficientSet, Ŕ;
14         Generate the RefSet consisting of b feasible solutions;
15         Select subsets of solutions in RefSet and apply the combination processes to those subsets;
16         Run the LocalSearchCompromise to the solutions obtained in the previous step;
17         Update RefSet;
18 until (a maximum number of iterations is reached);
19 return EfficientSet, Ŕ;
```

Figure 1. BIOSS general algorithm pseudo code (Melián-Batista et al., 2014)

5.3 Multi-objective ant colony algorithm
This method is used to solve some multi-objective routing problems whose nature is compatible with the ant colony algorithm. The main idea of this method is to define the types of pheromones in the algorithm according to the number of objective functions (Jozefowiez, Semet, & Talbi, 2008).

5.4 Variable Neighborhood Taboo Search (VNTS) Heuristic Algorithm
This algorithm uses several neighboring operators to escape the local answer. According to the proposition (16), in each iteration, this algorithm evaluates the function that is made up of the weighted sum of the objective functions. The symbol n is for the normalized value of the objective functions and the parameter P is for adjusting the curvature of the objective function, which is usually a small positive number (Janssens, Van Den Bergh, Sörensen, & Cattrysse,
By assigning different weights to the objective functions, the algorithm searches for different areas of the answer, which results in the production of a set of scattered and efficient answers, the decision-maker can choose the right answer from them. Figure (2) shows the quasi-general code of this algorithm.

\[
f(x) = [w_1 f_1^n(x)^p + w_2 f_2^n(x)^p + w_3 f_3^n(x)^p]^{\frac{1}{p}}, \quad w_1 + w_2 + w_3 = 1
\]

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6. Conclusion

As a single-purpose problem, HVRPTW is regarded as an NP-hard problem (Molina et al, 2014). Therefore, the multi-objectivity of this problem increases the difficulties of solving it. Implementing appropriate solution methods for this type of problem requires computer programming techniques. So, implementing solution methods are not in line with the objectives of the multi-objective decision-making lesson. Therefore, in this section, we propose and describe only a few methods to solve the problem raised in the project. Pareto analysis in practice illustrates the application of the 80/20 rule to prioritization. Imagine that you have just started your new career as a department head. Not surprisingly, you have a lot of problems to
solve. Ideally, you want to focus on solving the biggest problems. But how do you know what is the first problem you have to solve? Do some problems have a common root? Pareto analysis is a simple technique for prioritizing changes that cause problems. Using this approach, you can prioritize individual changes that will boost your situation.

**Conflict of Interest:** The authors declare no conflict of interest.

**REFERENCES**


