



A Comparative Study of the Equal-Weight Method and Hierarchical Risk Parity in Portfolio Construction

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Research Article

Abstract

Purpose: Portfolio optimization is a process in which the capital is allocated among the portfolio assets such that the return is maximized while the risk is minimized. Portfolio construction and optimization has long been an active research area in finance. For the portfolios with highly correlated assets, the performance of traditional risk-based asset allocation methods such as, the mean-variance (MV) method is limited because quadratic optimizers require an inversion of the covariance matrix of the portfolio to distribute weight among the portfolio assets.

Methods: A possible solution to the limitations of traditional risk-based asset allocation methods can be provided by a hierarchical clustering-based Machine Learning method because it uses hierarchical relationships between the covariance of assets in the portfolio to distribute the weight, and inversion of the covariance matrix is not required. A comparison of the performance of a simple non-optimization technique called the Equal-weight (EW) method to the two optimization methods, the Mean-variance method and the HRP method, which is a machine learning method, was conducted in this research.

Results: It was found that in terms of cumulative returns, the equal-weight method has outperformed several more sophisticated optimization techniques, the mean-variance method, and the HRP method. For most of the period, the Sharpe ratio of the HRP method was observed to be similar to the mean-variance method and equal-weight method.

Implications: This research supports the idea that HRP is a feasible method to construct portfolios with correlated assets because the performance of HRP is comparable to the performances of the traditional optimization method and the non-optimization method.

Keywords: Equal-weight method, Hierarchical Risk Parity method, Mean-variance method, Hierarchical clustering.

1. Introduction

Portfolio construction and allocation of investment funds have always been a challenging issue in the asset management industry. Predictive analytics using machine learning techniques can be used in improving asset allocation methodologies in portfolio construction. By identifying the patterns and trends in the historical data, machine learning techniques can be used to construct portfolios that are likely to outperform the market by predicting the future returns on the assets.

The mean-variance model is a quadratic optimization method that aims at finding the best ratio of return to risk. The quadratic programming method requires an inversion of the covariance matrix. The inversion of the covariance matrix is prone to large errors when the covariance matrix is ill-conditioned with a high condition number. In a portfolio with an increase in the number of correlated assets, the condition number of the covariance matrix increases, and eventually, the condition number becomes so large that a small numerical error can make the inverse matrix unstable (Lopez de Prado, 2016). The requirement of the inversion of the covariance matrix in quadratic programming methods makes them unreliable for practical applications.

These instability concerns could be addressed using a hierarchical clustering-based asset allocation technique called Hierarchical Risk Parity (HRP). Hierarchical Risk Parity (HRP) uses hierarchical relationships between the covariance of the assets to create a portfolio. HRP approach circumvents the issues related to covariance matrix forecasts because HRP does not require an inversion of the covariance matrix, which is a requirement for most of the traditional risk-based asset allocation methods. HRP is a fairly new approach and the practical implementations of HRP are still very scarce in portfolio construction. The naïve portfolio diversification method may be the preferred method for constructing portfolios when there is a lack of statistical information such as variances or correlations about the portfolio assets. Naïve portfolio diversification methods are often less mathematically intensive driven mostly by common sense and are instinctive. In naïve diversification, the asset classes are typically chosen at random, and the capital is invested among asset classes equally, thus the overall risk of the portfolio is reduced naturally and the requirement for weight allocation calculations using sophisticated mathematical models is not required. On the contrary, for the optimization methods mathematical programming is used to decide the weight allocation among the portfolio assets. The naïve diversification strategy does not consider the correlation among the assets in the portfolio as opposed to the Hierarchical Risk Parity (HRP) method. DeMiguel (2009) proposed that very simple basic models perform reasonably well because there does not exist a statistically significant difference between the naïve approach and optimization approaches. The equal-weight method is a type of naïve diversification strategy.

According to DeMiguel (2009), complex and sophisticated methods do not always lead to an optimal investment strategy because such optimization techniques are usually constrained by estimation errors, e.g. in the mean-variance method, the requirement of forecasting returns can lead to potentially large estimation errors. A great deal of study has been performed in reducing the input errors of the optimizers. E.g., Michaud and Robert (1998) proposed that estimation error can be reduced by resampling. Jorion (1985) has proposed the Bayes-Stein approach. In this approach, the expected returns are compressed towards to minimum-variance portfolio. In the mean-variance approach, Chow (1995) has applied a benchmark-tracking error term to augment the mean-variance objective function. Chevrier and McCulloch (2008) used Bayesian estimation for the optimization inputs.

According to the study of Kritzman et al. (2010), optimized portfolios outperform equal-weight portfolios. In their study optimized portfolios have generated superior out-of-sample performance compared to the equal-weight portfolios. The study concludes that to estimate expected returns small historical samples are often used; small sample size is a debatable assumption. The conception that the performance of an equal-weight portfolio is superior to an optimized portfolio arises from the reliance on sample historical samples. There are a few benefits to the equal-weight method which makes it favorable for constructing portfolios in financial institutions. The equal-weight method can mitigate the concentration bias because weight is equally distributed among all the assets of the portfolio. On the other hand, in the traditional mean-variance method, more weight is allocated to the assets with high returns, low volatility, or assets with negative correlation concerning returns. During rebalancing of the assets in the equal-weight method the strategy of "buy low, sell high" is efficiently followed by which expensive assets are sold and cheaper assets are purchased thereby the method invests in assets that perform the best. The method thus has the advantage of the mean-reversion effect. The equal-weight method does not underperform the asset which is the worst

performing. The method happens to invest in the best-performing assets. Because of smaller turnovers, the transaction cost in the equal-weight method is comparatively less compared to the other dynamic asset allocation methods. DeMiguel et al. (2009) have also suggested that out-of-sample performance for the equal-weight method is better than the other traditional asset allocation methods. The method has a high alpha (α) which is a measure of the excess return of an investment with respect to a benchmark index representing the market. There is empirical research evidence suggesting that in many instances the equal-weight model has outperformed many complex models (DeMiguel et al., 2009).

The diversification benefit, however, for the equal-weight method is low because the method is usually applied to assets with less available information about their characteristics such as correlation, volatilities, and returns. If some information about the expected returns, volatility, and diversification of the portfolio assets is available, optimization is probably the chosen method.

2. Literature Review

An equal-weight portfolio is an intuitive way to create a diversified portfolio when the capital is spread across the portfolio assets equally. Equal weight portfolio does not suffer from estimation errors. Researchers have observed that the equal-weight portfolio attains higher returns and volatility compared to the optimized portfolios (Lessard, 1976; Roll, 1981; Ohlson & Rosenberg, 1982). Korajczyk and Sadka (2004) have also observed that equal-weight portfolios have higher returns than the portfolios constructed using the optimization techniques.

There is empirical evidence showcasing the equal-weight portfolios outperforming the value-weighted and price-weighted portfolios (Plyakha et al., 2012; DeMiguel et al., 2009). DeMiguel et al. (2009) have found that on an equity portfolio, the equal-weight portfolio outperformed optimized portfolios with respect to the Sharpe ratio, certainty-equivalent return, or turnover. The study is convincing because 14 models were evaluated by the authors on 7 empirical datasets. Bayesian estimation has been used to reduce the estimation error in the study. It should be noted that despite the use of Bayesian estimation, none of the optimized portfolios consistently outperformed the equal-weight method. In the study, the Sharpe ratio of the equal weight method was 50% higher than the optimized mean-variance method (DeMiguel et al., 2009).

In an equal-weight portfolio, an equal monetary weight is placed on each asset of the portfolio irrespective of its market capitalization and the strategy requires frequent rebalancing. It provides a simple alternative to traditional optimization techniques (Bodie et al., 2014). Plyakha et al. (2012) observed that the equal-weight portfolio with monthly rebalancing outperformed an optimized portfolio even after considering the transaction costs. The difference in the performance of the two sets of portfolios was substantially attributed to monthly rebalancing. They have concluded that the higher mean return of the equal-weight portfolio was also due to its higher investment, size of the portfolio, and value factors. However, according to Monnier and Rulik (2011), the outperformance of the equal-weight portfolio can substantially be attributed to the monthly rebalancing strategy rather than to the difference in exposure to the market.

Rebalancing of the portfolio can be performed by different methods; however, it has been observed on simulated as well as historical data that a disciplined approach over a long period adds value by increasing returns or reducing risks (Arnott & Lovell, 1993; Plaxco & Arnott 2002; Buetow et al., 2002). When the transaction costs were taken into consideration, equal-weight portfolios did not outperform the optimized portfolios with a weekly rebalancing scheme. However, with a quarterly rebalancing scheme, the equal-weight portfolio outperforms the optimized portfolio (Grinblatt & Titman, 1989).

According to Statman (1987), when the number of randomly chosen equities in the portfolio is between 30 – 40 the equal-weight strategy has been shown to reduce the risk of the portfolio. Equal-weight portfolio has a 'buy low, sell high' attribute because the portfolio needs to be rebalanced to maintain equal weight. In rebalancing the appreciated assets are sold and the depreciated assets are purchased. Because of the

continuous requirement to rebalance an equal-weight portfolio, the method has an inherent advantage of selling the assets that have risen in value and purchasing the assets that have fallen following the previous rebalancing. This process tends to lock in gains as well as enhance the probability of exposure to the then cheaper stocks because of their previous underperformance. Thus, when returns are characterized by reversals in contrast to trends, equal-weight portfolios may offer better returns than non-equal-weighted portfolios. On the other hand, when the market is characterized by trends, optimized portfolios may be the investor's choice.

The equal-weight method is the preferred diversification strategy when minimal investment knowledge of the portfolio assets is available. Even if limited information about the asset's expected returns, volatilities, and diversification properties are available optimization strategies can be applied to improve a naively diversified portfolio (Kritzman et al., 2010).

3. Methodology

In an equal-weighted portfolio, all assets are allocated the same amount of investment weight, $\frac{1}{N}$, where N is the number of assets in the portfolio. For N number of assets in a portfolio, the weight on asset i could be expressed as,

$$w_i = \frac{1}{N}$$

The simple allocation rule for the equal weight strategy in transpose form can be expressed as,

$$X_{EW} = (1/N, \dots, 1/N)^T$$

The equal-weight method is a popular technique and is used broadly in the industry (Windcliff & Boyle, 2004). The hierarchical risk parity (HRP) method runs in three stages and uses the hierarchical relationship between the covariance of the assets in the portfolio to allocate weight among the assets in the portfolio (Palit & Prybutok, 2024).

The daily closing prices of the ETFs price data for the 9 sectors of the S&P 500 index were collected from January 31, 2000, to February 29, 2016. The assets of the portfolio are the US equities of 9 sectors of ETFs price data of the S&P 500 (Palit & Prybutok, 2024). The conclusion and findings in this research are not dependent upon the time range for which market data was collected.

The selected sector ETFs of the S&P 500 used to construct the portfolios are,

- Utilities (XLU)
- Materials (XLB)
- Technology (XLK)
- Industrials (XLI)
- Healthcare (XLV)
- Financials (XLF)
- Energy (XLE)
- Consumer Discretionary (XLY)
- Consumer Staples (XLP)

Three portfolios using the above-mentioned sector ETFs price data were constructed using three different methods namely the Hierarchical Risk Parity method (HRP) which is a hierarchical clustering-based machine learning method, the Equal-weight method ($1/N$), and the Mean-variance method (MV) which is an optimization technique.

4. Results

The performance of the equal-weight portfolio was compared to the performances of the two optimized portfolios constructed using the mean-variance method and HRP method. Each of the assets in the equal-weight method portfolio has an equal weight allocation of $1/9$. In figure 1 the daily returns of the equal-weight portfolio were presented.

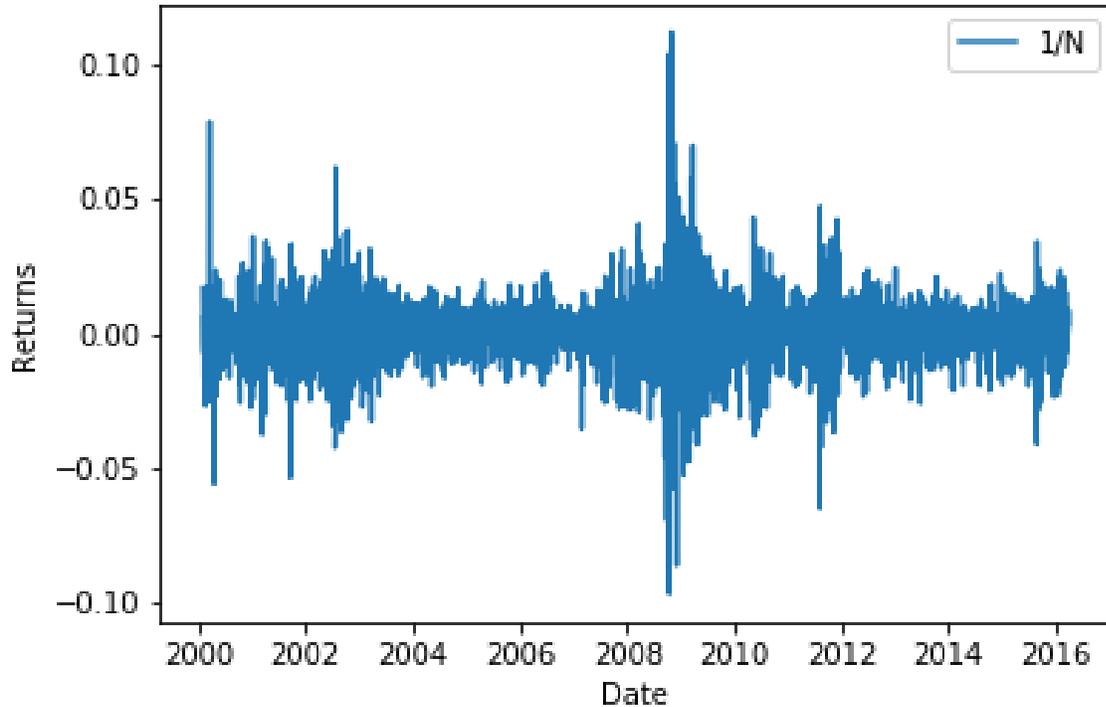


Fig. 1: Daily returns of the equal-weight portfolio

The cumulative returns of the mean-variance method, HRP method, and equal-weight method portfolios are presented in Figure 2. It can be observed that all three portfolios have the same tendency, and for most of the period, the equal-weight portfolio has greater cumulative returns than the cumulative returns of the portfolios created using the mean-variance method and HRP method. The cumulative returns of the HRP method were greater than the mean-variance method during most of the period (Palit & Prybutok, 2024). The rolling annual returns of the mean-variance method, HRP method, and the equal-weight method are presented in Figure 3. It can be observed that the rolling annual returns of the three portfolios are very comparable with similar trends.

The rolling annual volatility of the equal-weight method, HRP method, and the mean-variance method have been presented in Figure 4. It can be seen from Figure 4 that the volatility of the equal-weight portfolio has been higher than the other two portfolios for most of the time. The volatility of the HRP portfolio for most of the time has been less than the equal-weight portfolio and greater than the mean-variance portfolio. The trend in volatility for the three portfolios has been very similar.

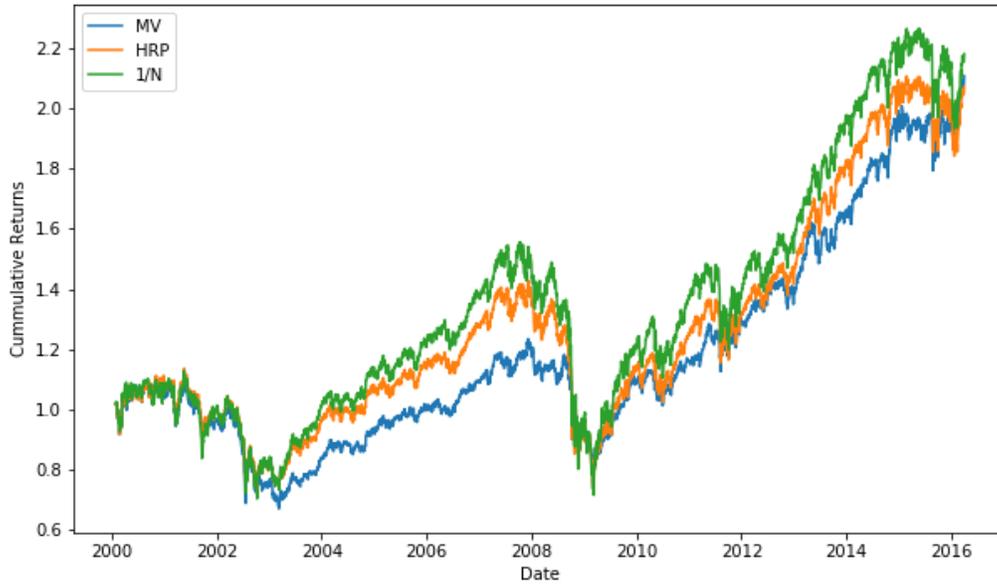


Fig. 2: Cumulative returns of the MV, HRP, and Equal-weight portfolio

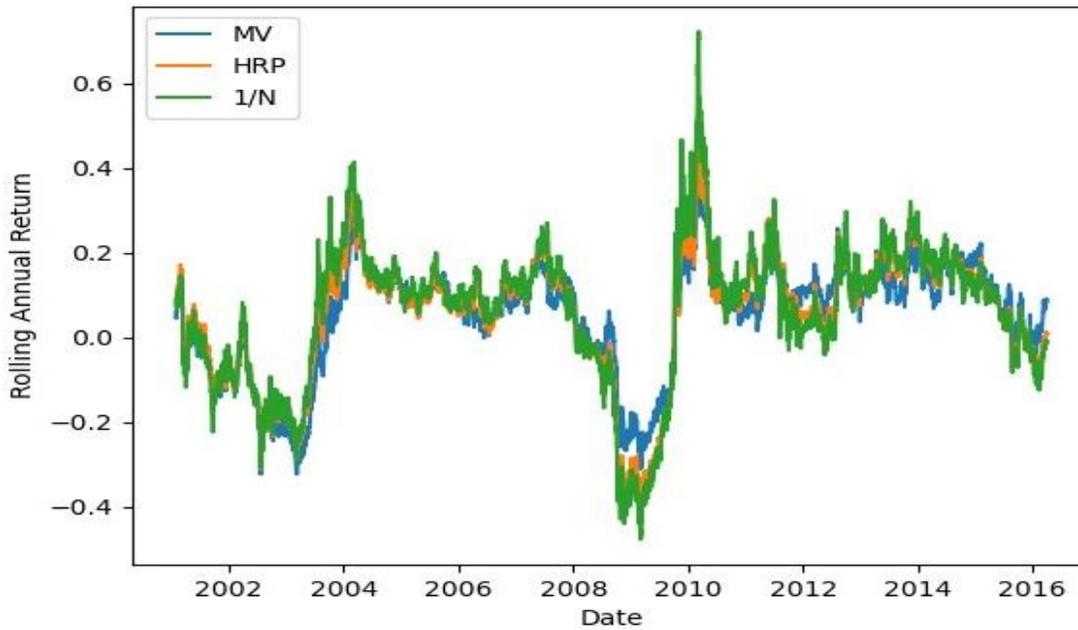


Fig. 3: Rolling annual returns of the MV, HRP, and Equal-weight portfolio

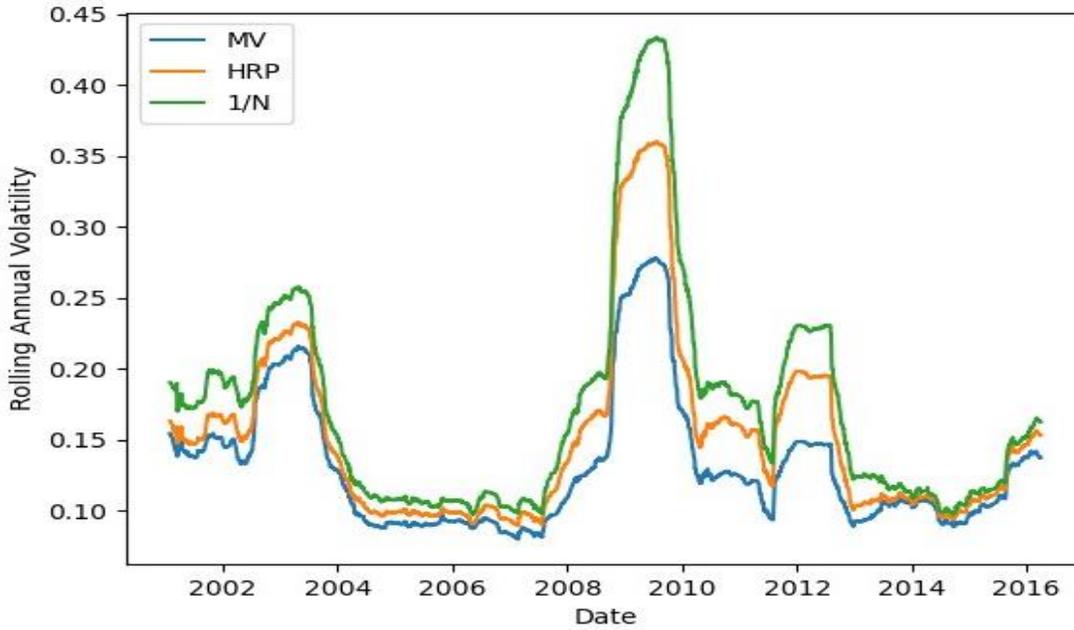


Fig. 4: Rolling annual volatility of the MV, HRP, and Equal-weight portfolio

The rolling Sharpe ratio of the equal-weight method, HRP method, and the mean-variance method have been presented in Figure 5. The rolling Sharpe ratio tells us about the performance of the portfolios. It can be seen from Figure 5 that there have been times when the performance of the mean-variance portfolio and the HRP portfolio have been better than the equal-weight portfolio, but overall, the rolling Sharpe ratio for the three portfolios has been very similar displaying a nearly similar trend.

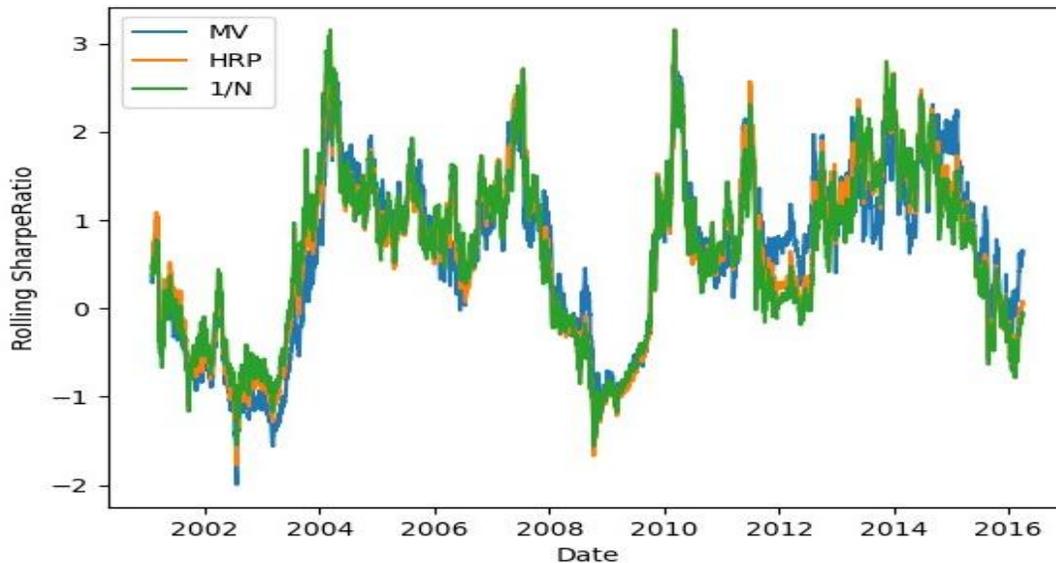


Fig. 5: Rolling Sharpe ratio of the MV, HRP, and Equal-weight portfolio

The correlation between the assets in the portfolio is a significantly decisive factor in portfolio construction and management governing the relationship between the assets of the portfolio. Correlations of the assets tell us how the price movements of the assets are related to each other. The assets with a perfectly positive correlation of 1, move in the same direction. The assets with a correlation of 0 move independently of each other with no predictable pattern. With a perfectly negative correlation of -1 between the assets, they move in opposite directions.

When the correlation of the assets lies between 0.5 – 1 the assets are said to have a strong positive correlation. The assets with a strong positive correlation enhance the chances of simultaneous gains or losses in the portfolio. The assets whose correlation lies between 0 – 0.5 have a relatively limited relationship. Such assets provide greater benefits of diversification. Assets with a correlation between -1 and 0 have a negative correlation and move in the opposite directions thereby providing a possible risk mitigation and diversification.

The average correlation between the assets of the portfolio over the period of analysis has been presented in Table 1. An average correlation of 0.6 has been observed among the assets of the portfolio.

Table 1: Average correlation between the assets of the portfolio

	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
XLB	1	0.66	0.7	0.79	0.65	0.58	0.5	0.62	0.72
XLE		1	0.53	0.6	0.5	0.47	0.51	0.49	0.52
XLF			1	0.79	0.7	0.65	0.53	0.68	0.79
XLI				1	0.77	0.66	0.54	0.72	0.83
XLK					1	0.56	0.47	0.67	0.75
XLP						1	0.57	0.64	0.67
XLU							1	0.52	0.5
XLV								1	0.7
XLY									1

Correlation values between the assets change over time. It is important to review the correlation values of the portfolio assets to maintain an optimal portfolio within the investment objective and risk tolerance of the investors.

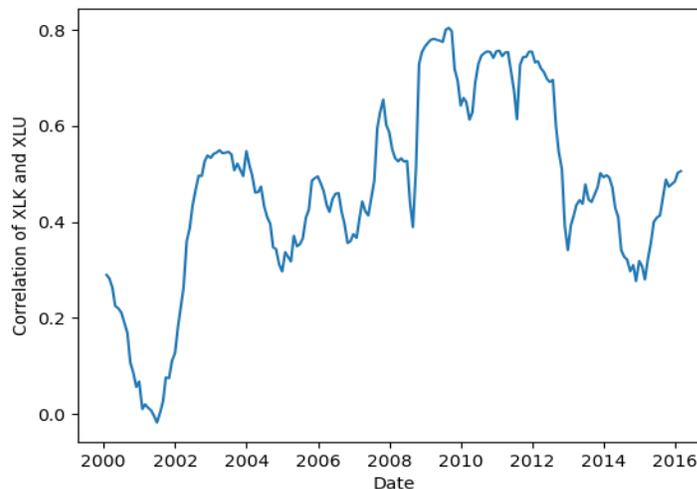


Fig. 6: Correlation between XLK and XLU

The lowest average correlation of 0.47 has been recorded between XLK and XLU. In Figure 6 it can be observed how the correlation between XLK and XLU has changed over time.

The highest average correlation of 0.83 has been recorded between XLI and XLY. Figure 7 presents the change in correlation between XLI and XLY. The average correlation of 0.6 has been recorded between XLI and XLE, which is also equal to the average correlation between the assets in the portfolio. Figure 8 presents the correlation between XLI and XLE. Figure 9 shows the correlation between XLP and XLV. The average correlation between XLP and XLV has been 0.64.

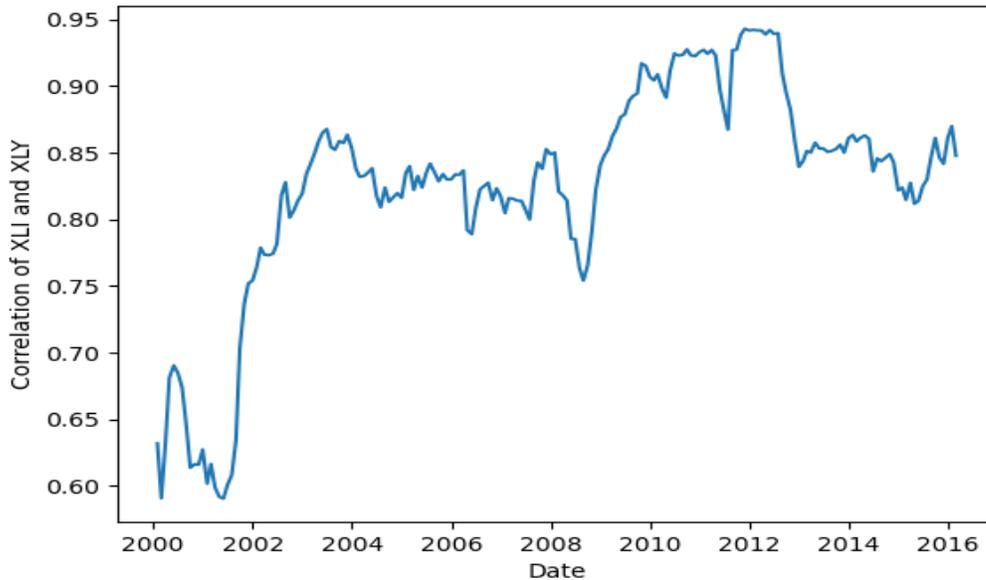


Fig. 7: Correlation between XLI and XLY

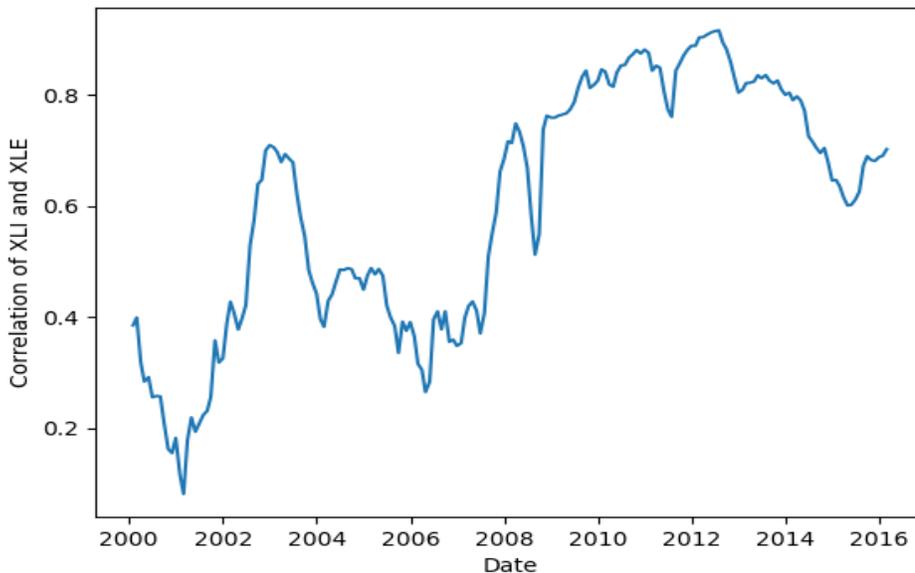


Fig. 8: Correlation between XLI and XLE

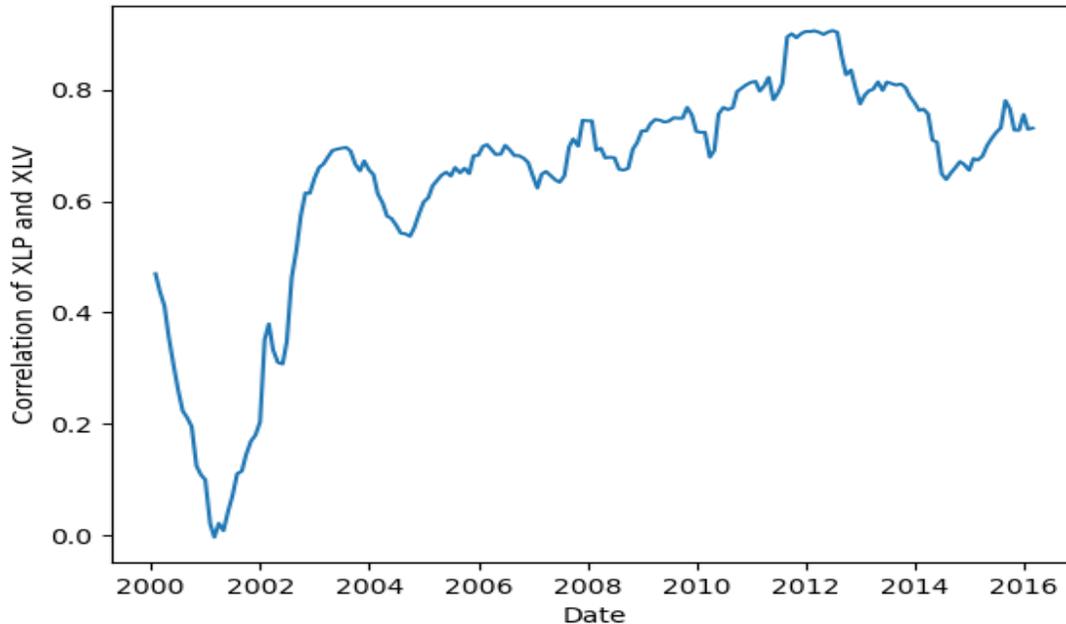


Fig. 9: Correlation between XLP and XLV

The performance of the equal-weight portfolio is very close to the performance of the HRP method and the mean-variance method.

Equal-weight portfolios require rebalancing over time to maintain equal weight on all assets in the portfolio. During each rebalancing, the system sells the appreciated assets and buys the depreciated assets; thus, following the mean reversion strategy. The mean reversion strategy states that the historical returns and asset price volatility eventually revert to the long-term mean or the average level of the entire dataset. The equal-weight method does not require an estimation of returns as an input and can enable diversification intrinsically. An investor using the equal-weight method for portfolio construction, thus, relies on the average correlation coefficient to estimate return-risk tradeoffs.

The optimization algorithms that are used to create non-equal weight portfolios are constructed with the idea of maximizing the expected returns for a chosen level of portfolio risk which depends on the standard deviations of returns and correlations between the assets of the portfolio. The equal-weight approach of portfolio construction allocates weights on the assets based merely on the number of assets present in the portfolio and does not consider any other factors. Like the optimization techniques, which are associated with the risk of input estimation; the equal-weight method likewise is associated with the risk of selecting the proper asset classes. The equal-weight method is a heuristic approach and does not provide the investors with the choice of risk tolerance. Nevertheless, portfolio rebalancing and weight allocation strategies are very critical decisions that an investor needs to decide upon which possibly determines the future performance and cumulative returns of a portfolio.

With an average correlation of 0.6, the assets in the portfolio of this research are not highly correlated. When the assets present in the portfolio are highly correlated the equal-weight portfolio and the mean-variance portfolio might run into risk concentration.

5. Discussion and Conclusion

In this study, traditional and hierarchical clustering-based techniques were used for constructing portfolios with US equities. The performance of the portfolios has been compared to analyze and compare the practicality and feasibility of constructing portfolios using the hierarchical clustering-based machine learning approach to the other traditional portfolio construction methods.

Among the traditional allocation methods, the mean-variance method and the equal-weight method were used for the study. Out of the many traditional asset allocation methods that are applicable, the above two traditional methods were chosen because the mean-variance method forms the foundation of the Modern Portfolio Theory, and the equal-weight method, despite its simplicity, has outperformed the more sophisticated optimized models on multiple occasions as was observed by many researchers (DeMiguel et al., 2009; Plyakha et al., 2012).

The result of this study shows that the equal-weight portfolio has a similar performance to the two optimized portfolios created using the mean-variance method and HRP method. It is concluded that the assets selected to construct the portfolios for this research are not highly correlated. When the portfolio contains assets with high correlation which is the case in many practical scenarios, the performance of the equal-weight method may be affected adversely because of the concentration of risk in the portfolio. The weight allocation by the mean-variance method might be erroneous on a portfolio with correlated assets because the mean-variance method requires an inversion of the covariance matrix to distribute the weight in the portfolio.

This research concludes that in practical situations for constructing a portfolio that may have correlated assets, the HRP method is a viable weight allocation technique because it does not require an inversion of the covariance matrix, and the chances of risk concentration are limited in the HRP portfolio because the HRP method uses the information present in the covariance matrix for weight distribution. Thus, HRP is a viable method for constructing portfolios with correlated assets. This research supports the idea that a machine-learning technique can be used in portfolio construction.

This research also supports the observation that the equal-weight strategy, which is a very simple non-optimization technique can have a comparable performance to the sophisticated optimized portfolio construction strategies.

Different asset allocation methods distribute the investment weight differently among the assets in the portfolio. The choice of weight distribution strategy is a critical decision that an investor needs to take which finally determines the future of the portfolio performance.

Author's contribution: Debjani Palit developed the idea, performed statistical analysis, and wrote the first draft. Victor R. Prybutok evaluated and refined the concept, managed the project, and edited the draft.

Conflict of Interest: The authors declare no conflict of interest.

REFERENCES

- Arnott, R. D., & Lovell, R. D. (1993). Rebalancing: Why? When? How Often? *The Journal of Investing*, 2(1), 5-10.
- Buetow Jr, G. W., Sellers, R., Trotter, D., Hunt, E., & Whipple Jr, W. A. (2002). The benefits of rebalancing. *Journal of Portfolio Management*, 28(2), 23-32.
- Bodie, Z., Kane, A., & Marcus, A. (2014). *EBOOK: Investments-Global edition*. McGraw Hill.
- Chevrier, T., & McCulloch, R. E. (2008). Using Economic Theory to Build Optimal Portfolios. Available at SSRN 1126596. <http://dx.doi.org/10.2139/ssrn.1126596>
- Chow, G. (1995). Portfolio Selection Based on Return, Risk, and Relative Performance. *Financial Analysts Journal*, 51, 54-60.
- DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus Naive Diversification: How inefficient Is the 1/n Portfolio Strategy? *Review of Financial Studies*, 22(5), 1915-1953. <http://dx.doi.org/10.1093/rfs/hhm075>
- Grinblatt, M., & Titman, S. (1989). Mutual Fund Performance: An Analysis of Quarterly Portfolio Holdings. *The Journal of Business*, 62(3), 393-416. <http://dx.doi.org/10.1086/296468>
- Jorion, P. (1985). International Portfolio Diversification with Estimation Risk. *The Journal of Business*, 58(3), 259-278.
- Korajczyk, R. A., & Sadka, R. (2004). Are momentum profits robust to trading costs? *The Journal of Finance*, 59(3), 1039-1082.
- Kritzman, M., Page, S., & Turkington, D. (2010). In defense of optimization: the fallacy of 1/N. *Financial Analysts Journal*, 66(2), 31-39.

- Lessard, D. R. (1976). World, country, and industry relationships in equity returns: implications for risk reduction through international diversification. *Financial Analysts Journal*, 32(1), 32-38.
- Lopez De Prado, M. (2016). Building diversified portfolios that outperform out-of-sample. *The Journal of Portfolio Management*, 42(4), 59-69.
- Michaud, R., & Robert, O. (1998). *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation*, Boston: Harvard Business School Press.
- Monnier, B., & Rulik, K. (2011). Behind the Performance of Equally Weighted Indices. *Ossiam Research Team*, 1-7. <https://www.wertpapier-forum.de/applications/core/interface/file/attachment.php?id=64584>
- Ohlson, J., & Rosenberg, B. (1982). Systematic Risk of the CRSP Equal-weighted Common Stock Index: A History Estimated by Stochastic-Parameter Regression. *The Journal of Business*, 55(1), 121-145.
- Palit, D., & Prybutok, V. R. (2024). A Study of Hierarchical Risk Parity in Portfolio Construction. *Journal of Economic Analysis*, 3(3), 106-125. <https://doi.org/10.58567/jea03030006>
- Plaxco, L. M., & Arnott, R. D. (2002). Rebalancing a global policy benchmark. *Journal of Portfolio Management*, 28(2), 9 – 22.
- Plyakha, Y., Uppal, R., & Vilkov, G. (2012). Why does an equal-weighted portfolio outperform value and price-weighted portfolios? Available at SSRN 2724535. <https://dx.doi.org/10.2139/ssrn.2724535>
- Roll, R. (1981). A possible explanation of the small firm effect. *The Journal of Finance*, 36(4), 879-888.
- Statman, M. (1987). How many stocks make a diversified portfolio? *Journal of Financial and Quantitative Analysis*, 22(3), 353-363.
- Windcliff, H. & Boyle, P. (2004). The 1/N pension plan puzzle. *North American Actuarial Journal*, 8(1), 32-45.



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